Solutions - Homework 1

(Due date: January 23rd @ 5:30 pm)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (30 PTS)

a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (14 pts)

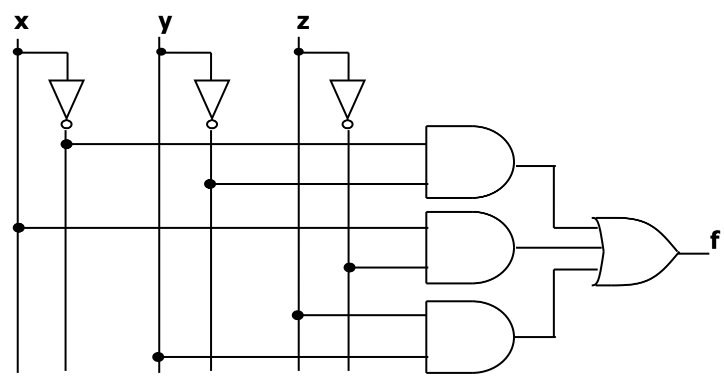
- ✓ $F = \overline{x\bar{y}z + y\overline{(x+z)}}$
- ✓ $F = \overline{A(\bar{B}\oplus\bar{C}) + \bar{B}}$
- ✓ $F = (\bar{B}+A)(\bar{C}+\bar{B})(C+A)+CA$
- ✓ $F(X,Y,Z) = \prod(M_1, M_3, M_6, M_7)$

✓ $F = \overline{x\bar{y}z + y\overline{(x+z)}} = \overline{y\overline{(x+z)} + x\bar{y}z} = \overline{y\overline{(x+z)}}.\overline{x\bar{y}z} = (x+\bar{y}+z)(\bar{x}+y+\bar{z})$

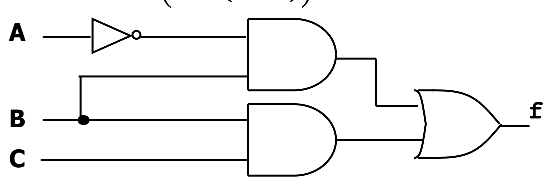
$F = x\bar{x} + xy + x\bar{z} + z\bar{x} + zy + z\bar{z} + \bar{y}\bar{x} + \bar{y}y + \bar{y}\bar{z} = xy + x\bar{z} + z\bar{x} + zy + \bar{y}\bar{x} + \bar{y}\bar{z}$

$F = x\bar{z} + \bar{x}\bar{y} + \bar{y}\bar{z} + xy + \bar{x}z + zy = x\bar{z} + \bar{x}\bar{y} + xy + \bar{x}z + zy$

$F = x\bar{z} + xy + yz + \bar{y}\bar{x} + \bar{x}z = x\bar{z} + xy + yz + \bar{y}\bar{x} = zy + \bar{z}x + xy + \bar{y}\bar{x} = zy + \bar{z}x + \bar{y}\bar{x}$

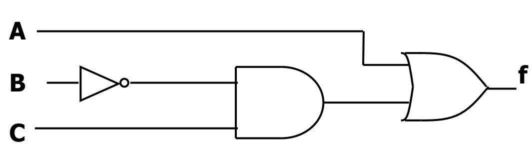


✓ $F = \overline{A(\bar{B}\oplus\bar{C}) + \bar{B}} = \overline{A(\bar{B}\oplus\bar{C})}.B = \left(\bar{A} + \left(\overline{\bar{B}\oplus\bar{C}}\right)\right).B = (\bar{A} + BC + \bar{B}\bar{C}).B = \bar{A}B + BC + B\bar{B}\bar{C} = \bar{A}B + BC$

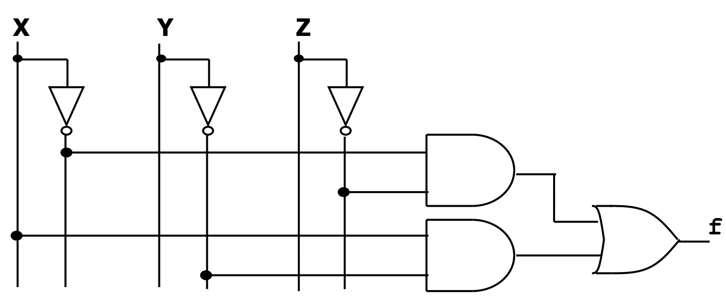


✓ $F = (C+A)(\bar{C}+\bar{B})(A+\bar{B}) + CA = (C+A)(\bar{C}+\bar{B}) + CA$

$(C+A)(\bar{C}+\bar{B}) + CA = C\bar{B} + \bar{C}A + A\bar{B} + CA = C\bar{B} + \bar{C}A + CA = C\bar{B} + A$



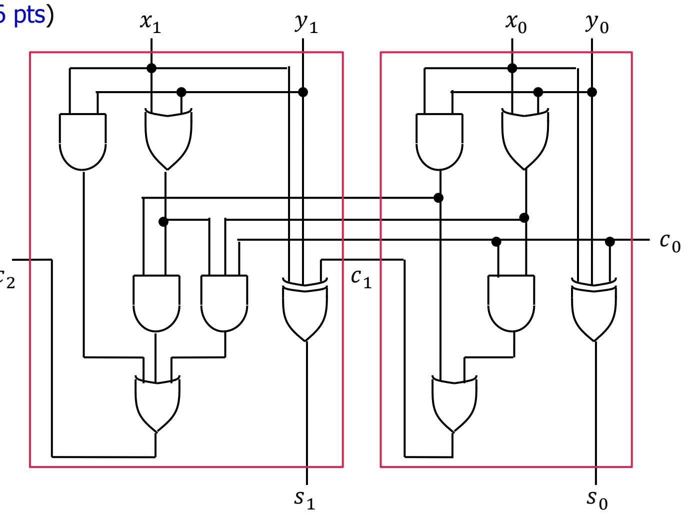
✓ $F(X,Y,Z) = \prod(M_1, M_3, M_6, M_7) = \sum(m_0, m_2, m_4, m_5) = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + X\bar{Y}Z = \bar{X}\bar{Z} + X\bar{Y}$



b) Given the following circuit with inputs x_0, x_1, y_0, y_1, c_0: (16 pts)

- Provide the Boolean expression (based on the circuit inputs) for s_0, s_1, c_1, c_2. (7 pts)

- For s_0 and c_1: (9 pts)
  - ✓ Express the Boolean functions using both the minterms and maxterms representations. (4 pts)
  - ✓ Provide the Boolean functions using the Canonical Sum of Products (SOP), and Product of Sums (POS).
  - ✓ Sketch the logic circuits as Canonical Sum of Products and Product of Sums. (3 pts)



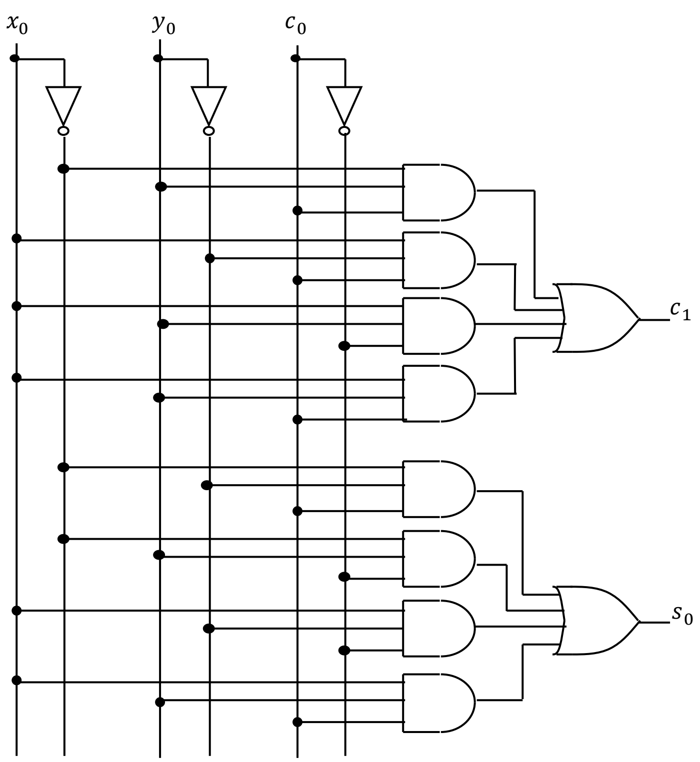
$s_0 = x_0 \oplus y_0 \oplus c_0$
$c_1 = x_0 y_0 + (x_0 + y_0) c_0 = x_0 y_0 + x_0 c_0 + y_0 c_0$
$s_1 = x_0 \oplus y_0 \oplus c_1 = x_0 \oplus y_0 \oplus (x_0 y_0 + x_0 c_0 + y_0 c_0)$
$c_2 = x_1 y_1 + (x_1 + y_1) x_0 y_0 + (x_1 + y_1)(x_0 + y_0) c_0$

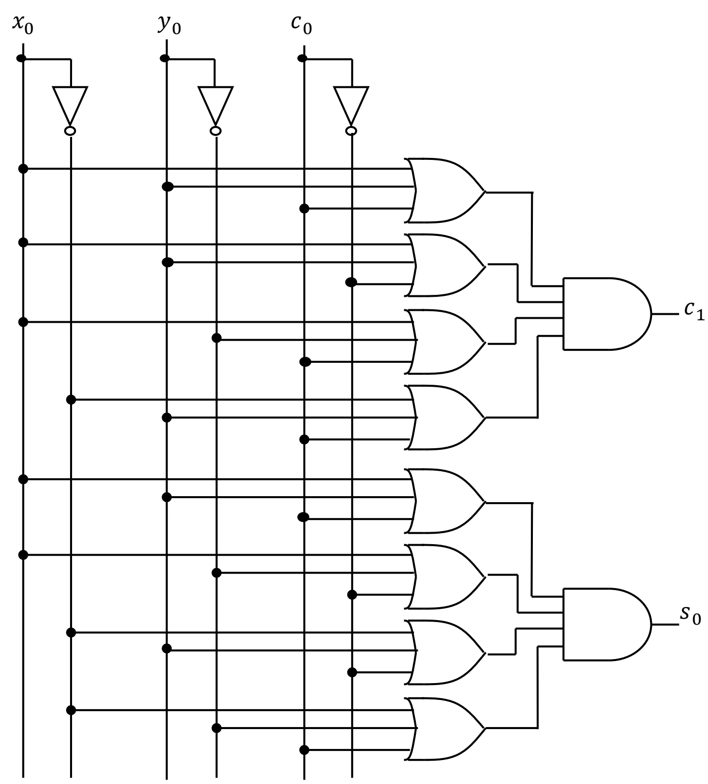
Minterms and maxterms:
$\Rightarrow c_1(x_0, y_0, c_0) = \sum(m_3, m_5, m_6, m_7) = \prod(M_0, M_1, M_2, M_4)$
$\Rightarrow s_0(x_0, y_0, c_0) = \sum(m_1, m_2, m_4, m_7) = \prod(M_0, M_3, M_5, M_6)$

x_0	y_0	c_0	c_1	s_0
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Sum of Products:
$c_1 = \overline{x_0} y_0 c_0 + x_0 \overline{y_0} c_0 + x_0 y_0 \overline{c_0} + x_0 y_0 c_0$
$s_0 = \overline{x_0}\,\overline{y_0} c_0 + \overline{x_0} y_0 \overline{c_0} + x_0 \overline{y_0}\,\overline{c_0} + x_0 y_0 c_0$

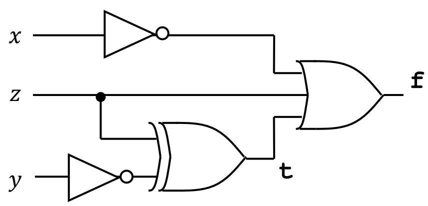
Product of Sums:
$c_1 = (x_0 + y_0 + c_0)(x_0 + y_0 + \overline{c_0})(x_0 + \overline{y_0} + c_0)(\overline{x_0} + y_0 + c_0)$
$s_0 = (x_0 + y_0 + c_0)(x_0 + \overline{y_0} + \overline{c_0})(\overline{x_0} + y_0 + \overline{c_0})(\overline{x_0} + \overline{y_0} + c_0)$





PROBLEM 2 (24 PTS)

a) Complete the truth table describing the output of the following circuit and write the simplified Boolean equation (6 pts).

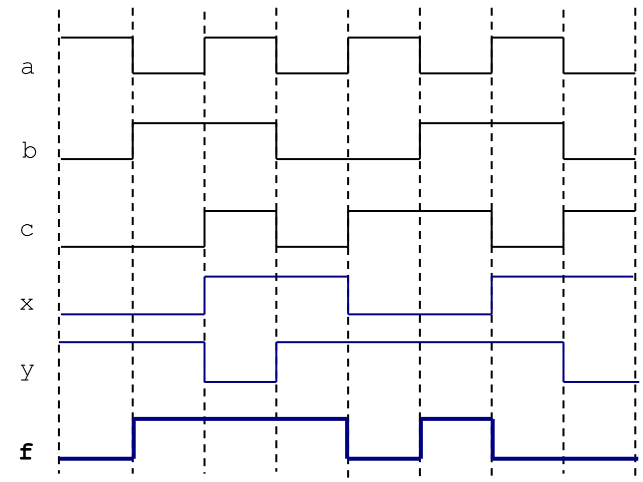


x	y	z	t	f
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	1
1	1	0	0	0
1	1	1	1	1

$f = \bar{x} + \bar{y} + z$

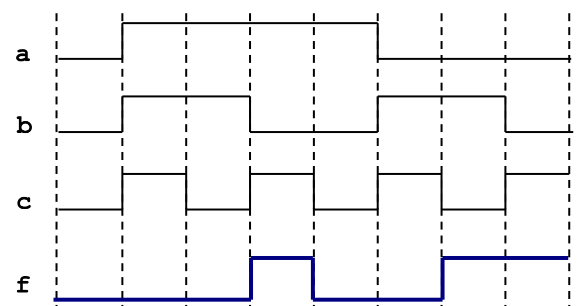
b) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (5 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
         f: out std_logic);
end circ;

architecture struct of circ is
  signal x, y: std_logic;

begin
  f <= y xnor (not a);
  x <= a xor (not b);
  y <= x nand c;
end struct;
```



c) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code. (8 pts)

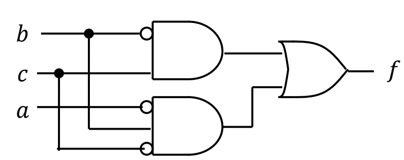
```
library ieee;
use ieee.std_logic_1164.all;

entity wave is
  port ( a, b, c: in std_logic;
              f: out std_logic);
end wave;

architecture struct of wave is
  signal x: std_logic;
begin
   x <= not(a) and b and not(c);
   f <= (not(b) and c) or x;

end struct;
```



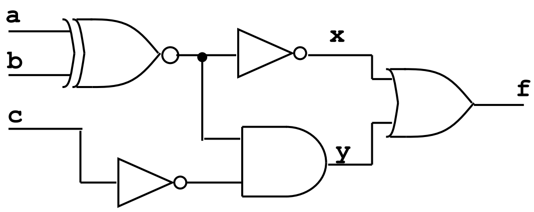
a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

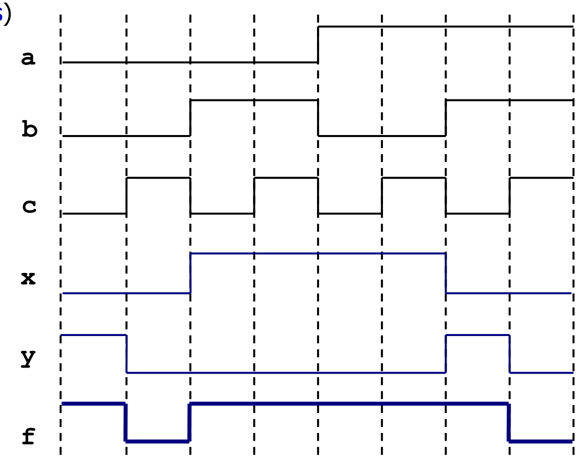
c \ ab	00	01	11	10
0	0	1	0	0
1	1	0	0	1

$f = \bar{b}c + \bar{a}b\bar{c}$



d) Complete the timing diagram of the following circuit: (5 pts)





PROBLEM 3 (10 PTS)

- A majority gate has an output value of 1 if there are more 1's than 0's on its inputs. The output is 0 otherwise.
- Design (provide the simplified Boolean equation and sketch the logic circuit) a 4-input majority gate with inputs a, b, c, d, and output f.

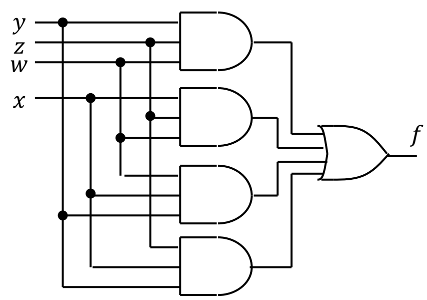
  The function break ties in favor of zeros when the number of inputs is even. Example: abcd=1100 → f = 0.

x	y	z	w	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

zw \ xy	00	01	11	10	
00	0	0	0	0	$\bar{z}\,\bar{w}$
01	0	0	1	0	$\bar{z}\,w$
11	0	1	1	1	$z\,w$
10	0	0	1	0	$z\,\bar{w}$
	$\bar{x}\,\bar{y}$	$\bar{x}\,y$	$x\,y$	$x\,\bar{y}$	

$\bar{x}$ x
$\bar{y}$ y $\bar{y}$
$\bar{z}$ z $\bar{w}$ w $\bar{w}$

$f = yzw + xzw + xyw + xyz$

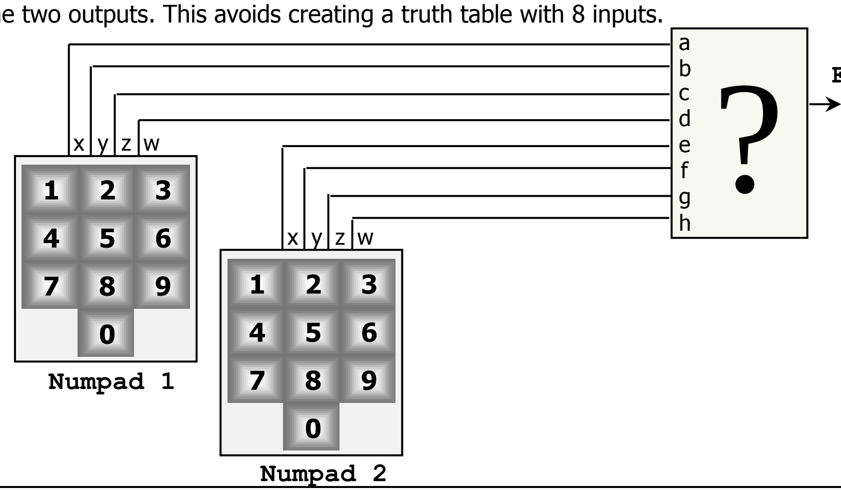


PROBLEM 4 (11 PTS)

- Design a logic circuit (simplify your circuit) that opens a lock ($f = 1$) whenever the user presses the correct number on each numpad (numpad 1: **9**, numpad 2: **4**). The numpad encodes each decimal number using BCD encoding (see figure). We expect that the 4-bit groups generated by each numpad be in the range from `0000` to `1001`. Note that the values from `1010` to `1111` are assumed not to occur.

  Suggestion: Create two circuits: one that verifies the first number (**9**), and another that verifies the second number (**4**). Then perform the AND operation on the two outputs. This avoids creating a truth table with 8 inputs.

BCD code x	y	z	w	Number pressed
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9



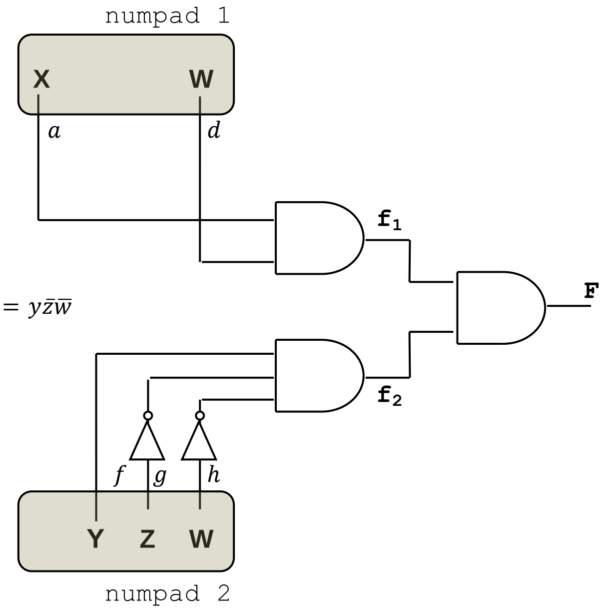
x	y	z	w	f_1	f_2
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	0	0
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	0	1	1	0
1	0	1	0	X	X
1	0	1	1	X	X
1	1	0	0	X	X
1	1	0	1	X	X
1	1	1	0	X	X
1	1	1	1	X	X

zw \ xy	00	01	11	10
00	0	0	X	0
01	0	0	X	1
11	0	0	X	X
10	0	0	X	X

$f_1 = xw$

zw \ xy	00	01	11	10
00	0	1	X	0
01	0	0	X	0
11	0	0	X	X
10	0	0	X	X

$f_2 = y\bar{z}\bar{w}$

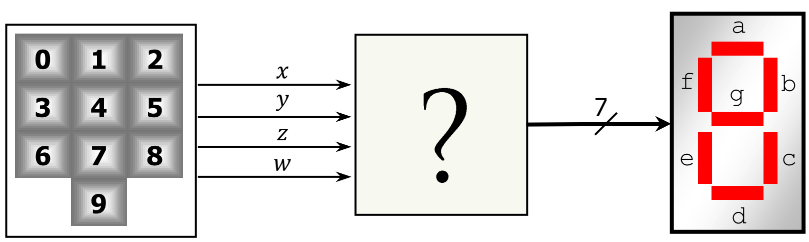


$F = ad\bar{f}g\bar{h}$

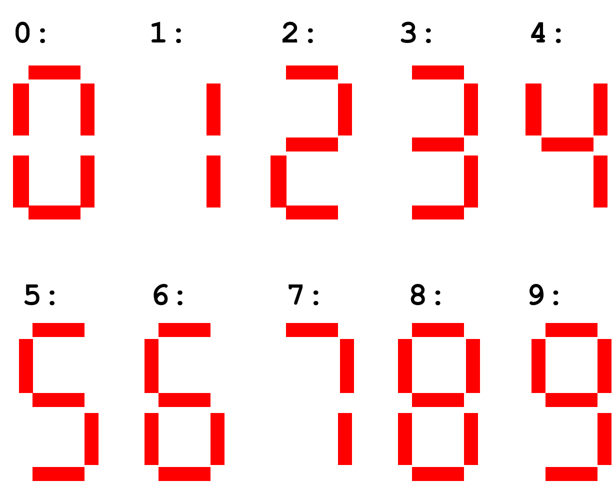
PROBLEM 5 (25 PTS)

- A numeric keypad produces a 4-bit code as shown below. We want to design a logic circuit that converts each 4-bit code to a 7-segment code, where each segment is an LED: A LED is ON if it is given a logic '1'. A LED is OFF if it is given a logic '0'.

  - ✓ Complete the truth table for each output (a, b, c, d, e, f, g).
  - ✓ Provide the simplified expression for each output (a, b, c, d, e, f, g). Use Karnaugh maps for a, b, c, d, e and the Quine-McCluskey algorithm for f, g. Note it is safe to assume that the codes 1010 to 1111 will not be produced by the keypad.

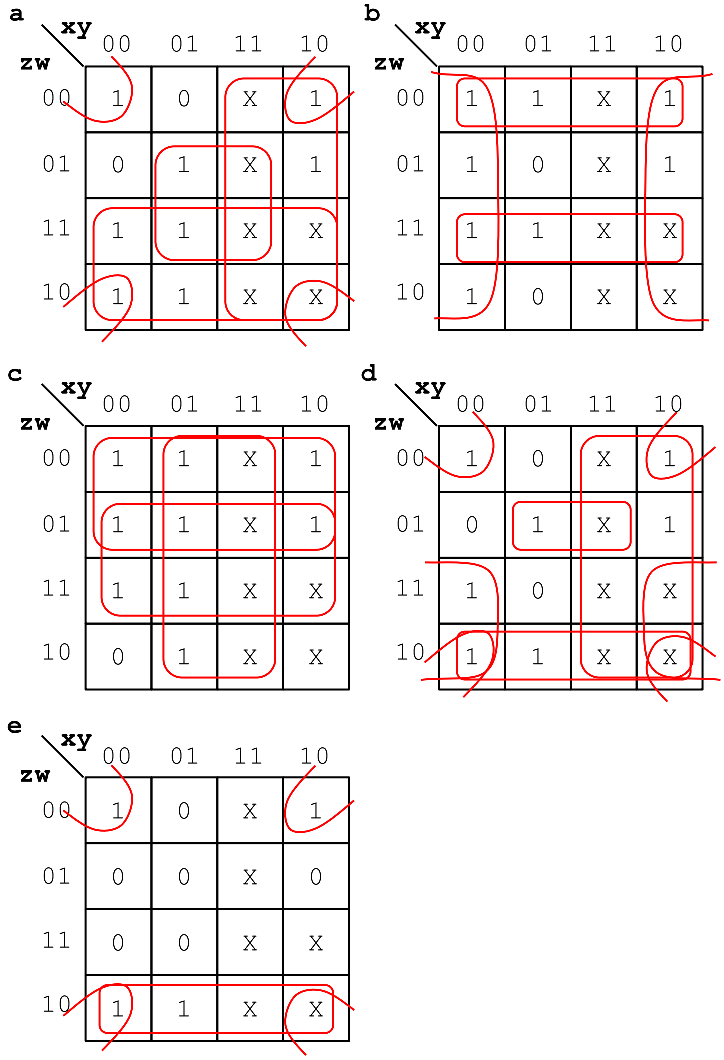


Value	x y z w	a b c d e f g
0	0 0 0 0	
1	0 0 0 1	
2	0 0 1 0	
3	0 0 1 1	
4	0 1 0 0	
5	0 1 0 1	
6	0 1 1 0	
7	0 1 1 1	
8	1 0 0 0	
9	1 0 0 1	1 1 1 1 0 1 1
	1 0 1 0	
	1 0 1 1	
	1 1 0 0	
	1 1 0 1	
	1 1 1 0	
	1 1 1 1	



Value	x y z w	a b c d e f g
0	0 0 0 0	1 1 1 1 1 1 0
1	0 0 0 1	0 1 1 0 0 0 0
2	0 0 1 0	1 1 0 1 1 0 1
3	0 0 1 1	1 1 1 1 0 0 1
4	0 1 0 0	0 1 1 0 0 1 1
5	0 1 0 1	1 0 1 1 0 1 1
6	0 1 1 0	1 0 1 1 1 1 1
7	0 1 1 1	1 1 1 0 0 0 0
8	1 0 0 0	1 1 1 1 1 1 1
9	1 0 0 1	1 1 1 1 0 1 1
	1 0 1 0	X X X X X X X
	1 0 1 1	X X X X X X X
	1 1 0 0	X X X X X X X
	1 1 0 1	X X X X X X X
	1 1 1 0	X X X X X X X
	1 1 1 1	X X X X X X X

$a = \bar{y}\bar{w} + wy + x + z$
$b = \bar{y} + \bar{z}\bar{w} + zw$
$c = y + \bar{z} + w$
$d = x + z\bar{w} + \bar{y}z + \bar{w}\bar{y} + \bar{z}wy$
$e = \bar{w}\bar{y} + z\bar{w}$



- $f = \sum m(0,4,5,6,8,9) + \sum d(10,11,12,13,14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
0	m_0 = 0000 ✓	$m_{0,4}$ = 0-00 ✓ $m_{0,8}$ = -000 ✓	**$m_{0,4,8,12}$ = --00** ~~$m_{0,8,4,12}$ = --00~~	**$m_{8,10,12,14,9,13,11,15}$ = 1---** ~~$m_{8,9,12,13,10,14,11,15}$ = 1---~~
1	m_4 = 0100 ✓ m_8 = 1000 ✓	$m_{4,5}$ = 010- ✓ $m_{4,6}$ = 01-0 ✓ $m_{4,12}$ = -100 ✓ $m_{8,9}$ = 100- ✓ $m_{8,10}$ = 10-0 ✓ $m_{8,12}$ = 1-00 ✓	**$m_{4,5,12,13}$ = -10-** ~~$m_{4,12,5,13}$ = -10-~~ **$m_{4,6,12,14}$ = -1-0** ~~$m_{4,12,6,14}$ = -1-0~~ **$m_{8,9,10,11}$ = 10--** ~~$m_{8,10,9,11}$ = 10-~~ **$m_{8,9,12,13}$ = 1-0-** ✓ ~~$m_{8,12,9,13}$ = 1-0-~~ **$m_{8,10,12,14}$ = 1--0** ✓ ~~$m_{8,12,10,14}$ = 1--0~~	
2	m_5 = 0101 ✓ m_6 = 0110 ✓ m_9 = 1001 ✓ m_{10}= 1010 ✓ m_{12}= 1100 ✓	$m_{5,13}$ = -101 ✓ $m_{6,14}$ = -110 ✓ $m_{9,11}$ = 10-1 ✓ $m_{9,13}$ = 1-01 ✓ $m_{10,11}$ = 101- ✓ $m_{10,14}$ = 1-10 ✓ $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{9,13,11,15}$ = 1--1** ✓ **$m_{10,14,11,15}$ = 1-1-** ✓	
3	m_{11}= 1011 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{11,15}$ = 1-11 ✓		
4	m_{15}= 1111 ✓			

$f = \bar{z}\bar{w} + y\bar{z} + y\bar{w} + x\bar{y} + x$

Prime Implicants		Minterms					
		0	4	5	6	8	9
$m_{0,4,8,12}$	$\bar{z}\bar{w}$	**X**	X			X	
$m_{4,5,12,13}$	$y\bar{z}$		X	**X**			
$m_{4,6,12,14}$	$y\bar{w}$		X		**X**		
$m_{8,9,10,11}$	$x\bar{y}$					X	**X**
$m_{8,10,12,14,9,13,11,15}$	x					X	**X**

$f = \bar{z}\bar{w} + y\bar{z} + y\bar{w} + x$

- $g = \sum m(2,3,4,5,6,8,9) + \sum d(10,11,12,13,14,15)$.
  Too many minterms. We better optimize: $\bar{g} = \sum m(0,1,7) + \sum d(10,11,12,13,14,15)$

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
0	m_0 = 0000 ✓	**$m_{0,1}$ = 000-**		
1	**m_1 = 0001**			
2	m_{10}= 1010 ✓ m_{12}= 1100 ✓	$m_{10,11}$ = 101- ✓ $m_{10,14}$ = 1-10 ✓ $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{10,14,11,15}$ = 1-1-** ~~$m_{10,11,14,15}$ = 1-1- ✓~~ **$m_{12,14,13,15}$ = 11--** ~~$m_{12,13,14,15}$ = 11-- ✓~~	
3	m_7 = 0111 ✓ m_{11}= 1011 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	**$m_{7,15}$ = -111** $m_{11,15}$ = 1-11 ✓ $m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

$\bar{g} = \bar{x}\bar{y}\bar{z}w + \bar{x}\bar{y}\bar{z} + yzw + xz + xy$

Prime Implicants		Minterms		
		0	1	7
m_1	$\bar{x}\bar{y}\bar{z}w$		X	
$m_{0,1}$	$\bar{x}\bar{y}\bar{z}$	**X**	X	
$m_{7,15}$	yzw			**X**
$m_{10,14,11,15}$	xz			
$m_{12,14,13,15}$	xy			

$\bar{g} = \bar{x}\bar{y}\bar{z} + yzw \quad \Rightarrow \quad g = (x + y + z)(\bar{y} + \bar{z} + \bar{w})$